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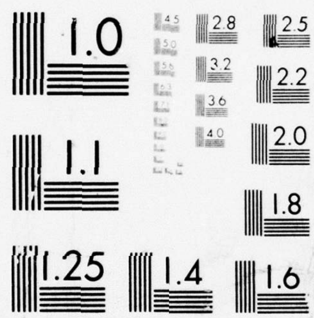

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MEMORANDUM REPORT NO. 2778

ESTIMATION OF PRECISION OF MEASUREMENT AND PRODUCT VARIABILITY: A COMPUTER MODEL

James F. O'Bryon

August 1977

USA ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
USA BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
MEMORANDUM REPORT NO. 2778

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER
(9)

4. TITLE (and Subtitle)
(6) ESTIMATION OF PRECISION OF MEASUREMENT AND PRODUCT VARIABILITY: A COMPUTER MODEL.

5. TYPE OF REPORT & PERIOD COVERED
BRL Memorandum Report.

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
(10) James F. O'Bryon

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
US Army Ballistic Research Laboratory
Aberdeen Proving Ground, MD 21005

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
(16) 1L662618AH80

11. CONTROLLING OFFICE NAME AND ADDRESS
US Army Materiel Development & Readiness Command
5001 Eisenhower Avenue
Alexandria, VA 22333

12. REPORT DATE
(11) AUGUST 1977

13. NUMBER OF PAGES
40

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
(12) 36p.

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

D D C
RECEIVED
SEP 7 1977
C

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
(14) BRL-MR-2778

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Precision	Statistics
Measurement Precision	FORTRAN
Bias	T-TEST
Product Variability	Variance
Multiple Instrumentation	Computer Program

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (ik)

When two or more instruments are used to simultaneously measure the same phenomena, it is possible, with a few assumptions, to obtain the most efficient unbiased estimator for the precision of measurement of each instrument as well as an estimation of the true variability of the event being observed. A computer program written in FORTRAN IV which performs these calculations is presented and discussed. Up to 10 instruments taking simultaneous measurements can be handled for a series of up to 50 observations per instrument. Provision is also made to determine whether any given instrument (Continued on back)

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is performing significantly different statistically from other instruments measuring simultaneously. Sample examples are also included to illustrate the program's usage and flexibility.

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I. INTRODUCTION

When one is taking measurements with the same instrument of successive occurrences of the same phenomena, one can then compute the variance and mean of the sample. If the measuring instrument has some sort of measurement error, this error becomes part and parcel of the sample's statistics unless one can somehow "strip out" this error from the real variability (product variability) and real mean of the phenomena being measured. This task is complicated further if the phenomena are non-replicatable as is the case with destructive testing.

When, however, two or more instruments independently and simultaneously measure successive occurrences, it is possible to estimate both the true product variability and the precision (or imprecision) of measurement of each instrument used. These methods have been published by Grubbs.[1]

These methods were later expanded to the case where one wishes to determine whether a given measuring instrument is operating as consistently as two other known instruments.[2] The nomenclature in this report is consistent with that found in the referenced reports.

A minimum of two instruments simultaneously measuring is needed. If three or more instruments are used, estimates of precision of measurement can be made which are totally free of the variability of the phenomena observed. This results in more direct comparision of the errors of measurement of each instrument. The procedures discussed here involve:

(1) The estimation of the precision of measurement,

(2) Estimation of product variability and,

(3) Significance tests for comparing the precisions of measurement of N instruments ($2 \leq N \leq 10$) and whether a given instrument has a bias which is so large compared to that of the other measuring devices that the instrument needs calibration.

---

[1] *Grubbs, F.E., "On Estimating Precision of Measuring Instruments and Product Variability," Journal of the American Statistical Association, Vol. 43, pp 243-264, June 1948.*

[2] *Grubbs, F.E. and O'Bryon, J.F., "The Statistical Comparison of Measuring Instrumentation," BRL ARDC Technical Report No. 11 (AD 732 428), July 1971.*

II. STATISTICAL OVERVIEW

A. Statistical Assumptions

The following statistical overview is reproduced from References 1 and 2 to enable the computer program user the opportunity to understand the functioning of the program. References are made in the examples to equations discussed here. For a comprehensive understanding of the statistical basis for the program, the reader is encouraged to study the referenced publications.

We shall first define some basic terminology. Below we see tabulated pairs of data points measured simultaneously by two instruments, I_1 and I_2,

Measurements by I_1	Measurements by I_2
$x_1 + e_{11}$	$x_1 + e_{12}$
$x_2 + e_{21}$	$x_2 + e_{22}$
.	.
.	.
.	.
$x_i + e_{i1}$	$x_i + e_{i2}$
.	.
.	.
.	.
$x_n + e_{n1}$	$x_n + e_{n2}$

where x_i = the true value of the characteristic considered, where i represents the ith item in the sample,

e_{ij} = the error in measurement of the ith item by the jth instrument, I_j.

It is assumed that both the characteristic being measured and the errors of measurement are normally distributed and that the absolute values of the characteristic measured and the errors of measurement are statistically independent, i.e.,

$$E(x_i^p \cdot e_{ij}^q) = E(x_i^p) \cdot E(e_{ij}^q).$$

In addition, it is assumed that the errors of measurement of the different instruments are statistically independent of each other, i.e.,

$$E(e_{ij}^{\,p} \cdot e_{\ell k}^{\,q}) = E(e_{ij}^{\,p}) \cdot E(e_{\ell k}^{\,q}) \qquad j \neq k \,.$$

It is also assumed that the errors of measurement are significantly smaller than the real value of the characteristic being measured.

B. General Case: Estimation of Precision of Measurement and Product Variability:

The basic way in which the computer program is used is to estimate the precision of measurement and product variability given observations from 2 to 10 instruments simultaneously.

Since the statistical treatment is somewhat different for the two instrument case, this statistical summary is presented in two parts.

(1) Two Instrument Case. For this case, there are two methods for estimating precision of measurement.

We take as an estimate of the variance in errors of measurement of instrument I_1,

$$\text{est.}\ (\sigma_{e1}^{\,2}) = S^2_{x+e1} - S_{x+e1,x+e2} \qquad (1)$$

where

$$S_{x+e1,x+e2} = \frac{1}{n-1} \sum_{i=1}^{n} \left\{(x_i + e_{i1}) - (\bar{x} + \bar{e}_1)\right\} \left\{(x_i + e_{i2}) - (\bar{x} + \bar{e}_2)\right\}$$

$$= \frac{1}{n(n-1)} \left\{ n \sum_{i=1}^{n} (x_i + e_{i1})\,(x_{2i} + e_{i2}) \right.$$

$$\left. - \left[\sum_{i=1}^{n} (x_i + e_{i1})\right] \left[(\sum_{i=1}^{n} (x_i + e_{i2})\right] \right\}$$

and as an estimate of the variance in errors of measurement of instrument I_2, we compute

$$\text{est.}\ (\sigma_{e2}^{\,2}) = S^2_{x+e2} - S_{x+e1,x+e2} \qquad (2)$$

and

$$E(S_{e1}^2) = \sigma_{e1}^2$$

$$E(S_{e2}^2) = \sigma_{e2}^2$$

where the estimates of the covariances

$$E(S_{x,e1}) = E(S_{x,e2}) = E(S_{e1,e2}) = 0 \text{ under the assumptions.}$$

It is remarked in passing that a technique of working with differences in corresponding measurements of the two instruments is of some interest. That is, if the measurements of I_2 are subtracted from corresponding measurements of I_1, the result gives

$$e_{11} - e_{12}$$

$$e_{21} - e_{22}$$

$$\vdots$$

$$e_{i1} - e_{i2}$$

$$\vdots$$

$$e_{n1} - e_{n2}$$

thus eliminating the true value of x_{ij}, the characteristic measured. Using the above differences, an estimate of σ_{e1}^2 is given by

$$\text{est. } (\sigma_{e1}^2) = \frac{1}{2}\{S^2_{x+e1} - S^2_{x+e2} + S^2_{e1-e2}\} . \qquad (3)$$

It is easy to see that estimate (3) is

$$S^2_{e1} + S_{x,e1} - S_{x,e2} - S_{e1,e2}$$

and hence precisely that given by (1).

In order to estimate the amount of variability in the product being measured, we may first add corresponding measurements of the two instruments, obtaining

$$\begin{array}{c} x_1 + e_{11} + x_1 + e_{12} \\ \cdot \\ \cdot \\ \cdot \\ x_i + e_{i1} + x_i + e_{i2} \\ \cdot \\ \cdot \\ \cdot \\ x_n + e_{n1} + x_n + e_{n2} \end{array}$$

The variance of the above sums of readings is

$$4S^2_x + S^2_{e1} + S^2_{e2} + 4S_{x,e1} + 4S_{x,e2} + 2S_{e1,e2} .$$

An estimate of the variability in the product, σ^2_x, is then given by

$$\text{est. } (\sigma^2_x) = \frac{1}{4} \{S^2_{x+e1+x+e2} - S^2_{e1-e2}\}, \tag{4}$$

The estimate of σ^2_x is given simply by the covariance of the readings of I_1 and I_2:

$$\text{est. } (\sigma^2_x) = S_{x+e1,x+e2} \tag{4a}$$

(Equation 4 and 4a are identical).

Estimate (4) or (4a) is unbiased and can be shown to be the maximum likelihood estimate.

The variance of the estimate, est. (σ^2_{e1}) or est. (σ^2_{e2}), depends on (A) σ^2_x, the variance in the characteristic measured, (B) σ^2_{e1}, the variance

of the errors of measurement of Instrument I_1, (C) σ^2_{e2}, the variance of errors of measurement of Instrument I_2 and (D) n, the number of observations or the sample size. Therefore, in order to obtain a precise estimate of σ^2_{e1} when using only two instruments, the variation in the characteristic measured, i.e., σ^2_x, should be held to a reasonable minimum or the sample size, n, should be sufficiently large.

Sometimes the estimated variance in errors of measurement may turn out to be negative. This may be due to small sample size, bad observations, errors in recording, outlying observations, etc. Since it is physically impossible for a true variance in errors of measurement to be negative, one suggestion is to add the absolute value of the negative variance to the estimated variance in error for the other instrument for an overall estimate of instrument error. The computer program prints negative as well as positive variances and stores these values to compute pooled estimates of variance in precision error for each instrument if multiple cases are input back-to-back using the same instruments. However, to avoid negative square roots, the program lists $\sigma_e = 0$ when taking the square root of a negative variance. See References 1, 2 and 4 for further information on the treatment of negative estimates of variance.

If the variation in the characteristic measured is zero (or if we measure the same item over and over again) i.e., if $\sigma^2_x = 0$, then one could compute

$$\frac{1}{n-1} \sum_{i=1}^{n} (e_{i1} - \bar{e}_1)^2 \tag{4b}$$

directly, and this would give an estimate of σ^2_{e1} with variance equal to

$$\frac{2}{n-1} \sigma^4_{e1} \quad . \tag{4c}$$

Apparently, in employing two instruments, there are only two computational procedures of interest for separating the variability in the product from the variance in the errors of measurement and both methods give the same estimate. In using either method, however, it is possible to estimate σ^2_{e1}, σ^2_{e2} and σ^2_x and thus determine from the relative order

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[4]*Thomson, W.A., Jr., "The Problem of Negative Estimates of Variance Components," Annals of Mathematical Statistics; Vol 33, pp 273-288, 1964.*

of magnitude of these quantities whether the instruments are sufficiently precise to use in taking the required measurements.

(2) Three or More Instrument Case ($3 \leq N \leq 10$).* Since the nomenclature has been established in the two instruments, this description of the $N \geq 3$ case will be limited to the basic equations solved by the computer program. As was stated earlier, using three or more instruments permits precision of measurement estimates totally free of the product variability itself.

The following equation is used to estimate precision of measurement of Instrument I_1 ($2 \leq i \leq 50$ data points)

$$\text{est. } (\sigma^2_{e1}) = S^2_{x+e1} - \frac{2}{N-1} \left\{ \sum_{r=2}^{N} S_{x+e1,x+er} \right\} + \frac{2}{(N-1)(N-2)} \left\{ \sum_{2\leq j<k}^{k=N} S_{x+ej,x+ek} \right\} . \qquad (5)$$

Estimates of all other instruments can be made by obvious rotations of the subscripts. The real product variability (stripped of measurement error) is estimated

$$\text{est. } (\sigma^2_x) \approx \frac{2}{N\,(N-1)} \sum_{1\leq r<s}^{s=N} S_{x+er,x+es} \qquad (6)$$

C. Comparative Case: Determination of Relative Performance of Two "Known" Instruments Versus One "Unknown" Instrument.

The application considered here is that of comparing an instrument of unknown performance characteristics with those of two known or proven test instruments to determine if their performance is statistically different.

If we measure a series of occurrences with three instruments simultaneously, it can be seen that the difference in readings of any two of the instruments consists of only the differences in (1) errors of measurement and (2) constant biases for the two instruments, and is hence free of the level of parameter measured. The model used here is based on this concept. From the variation or variance of the differences in errors of measurement from point to point of pairs of the three instruments making simultaneous

N is limited to 10 and n to 50 only because of the dimensions of the program's storage. The equations, of course, are valid for any $N \geq 3$, $n \geq 2$.

measurements, one can strip out and hence estimate the variance in errors of measurement of each of the three instruments. These figures therefore are basic to determining whether the precision of measurement is suitably good as compared with an estimate of the variance of true readings in order to judge precision of measurement properly. This, along with a comparison of levels of readings measured by the instrument, will determine accuracy or the need for calibration. Moreover, statistical tests of significance for precision and accuracy of measurement are rather easily developed and applied, as will be seen.

The statistical or mathematical model used for the observed measurements of the three instruments is indicated by the quantities.

$$\beta_1 + x_i + e_{i1} = R_i \qquad \beta_2 + x_i + e_{i2} = S_i \qquad \beta_3 + x_i + e_{i3} = T_i$$

where β_1, β_2 and β_3 are respectively the unknown instrumental biases, e_{i1}, e_{i2} and e_{i3} the random errors of measurement of the three instruments, and x_i the true unknown reading of the ith data point or characteristic measured in a series of measurements on n data points.

In order to estimate the standard deviation of the true readings measured, one may use the following formula which gives an unbiased estimate of the true reading variance

$$\text{est. } (\sigma_x^2) = (S_{x+e1,x+e2} + S_{x+e2,x+e3} + S_{x+e1,x+e3})/3 \qquad (7)$$

where the S's are sample covariances of the three instruments, taken two at the time, each based on n-1 degrees of freedom.

For the three instrument case, variance in precision error for first instrument is

$$\text{est. } (\sigma_{e1}^2) = S_{x+e1}^2 + S_{x+e2,x+e3} - S_{x+e1,x+e2} - S_{x+e1,x+e3} \quad . \qquad (7a)$$

Again estimates for the other instruments are made by simply "rotating the subscripts" of the above equation. We next wish to determine whether the two "known" instruments are performing consistently well.

Two Student t-tests are outlined which help to determine whether the two standard instruments chosen are performing consistently well to enable them to make a reliable **judgment** concerning the performance of a third unknown instrument submitted for study.

The first test examines the two standards for relative precision. Definition of terms used in the following equations are described in **Reference 2.** These instruments should be roughly equivalent in precision to be used as a basis of comparison. If this t-test shows no significant difference in precision of measurement, the levels of precision should still be examined to assure that the error of precision is much less than the actual parameter variation itself. The test takes the following form

$$t = \frac{r(yz)\sqrt{n-2}}{\sqrt{1-r^2(yz)}} \quad . \tag{8}$$

We should now define some more terms in the order presented,

y = the difference between the sum of mean readings of the two standard instruments and the sum of the ith readings of the same instruments,

$z = v_1$ = the difference in readings between the two standard instruments,

$r(yz)$ = the sample correlation coefficient of y and z,

$S(z)$ = the standard deviation of the z sample,

u_1 = the difference between the test instrument reading and the average of the readings of the two standards.

The second test (Equation 9) looks at the same two instruments for average level of readings. If the t-test detects a significant difference, recalibration should be performed to bring the instruments into better agreement

$$t_o = \bar{z}\sqrt{n}/S(z) \ . \tag{9}$$

We might say here that occasionally this t-test (Equation 9) for average levels detects a significant difference even for very small differences in average readings. This is caused usually by the influence of the very small standard deviation in the difference in the readings of the two known instruments over the series of occurrences.

Once both t-tests have been passed, it can be established that the standard instruments are performing consistently with each other.

Since one many times has no way of knowing the real data values, (i.e., destructive testing) the assumption is made that the best estimate of the real value is midway between the two standard measurements. Hence,

the average readings from these two standard instruments are computed and compared with the observations gathered from the unknown instrument submitted for study.

Tests for precision and bias (similar to those just outlined to determine a valid test) are performed, this time comparing the average of the standard readings with readings from the third instrument.

$$t_o\ (n-2) = \frac{[S^2(u_1)S^2(v_1) - .75]\sqrt{n-2}}{[3(1-r^2(u_1v_1))(S^2(u_1)S^2(v_1))]^{1/2}} \tag{10}$$

If the t-test for precision detects a significant difference (Equation 10) one should determine why the test showed significance. Mere failure of the test does not constitute an inferior performance by the third instrument. If the estimate of error in precision of the unknown instrument (T) is actually much less than the average of the two standards, the third instrument has an acceptable precision, even though the t-test showed significance. In fact its precision appears to be superior to the standards.

Finally, the levels of measurement of the unknown instrument are compared with the average readings from the standard instruments.

$$t_o\ (n-1) = \bar{u}_1\sqrt{n}/S(u_1) \tag{11}$$

This statistical procedure is flexible enough to use with any number of instruments and not limited to the three described above. Applying these tests enables those interested to generate a "data bank" of typical instrument performance.

It is possible, after groups of data points are gathered, to combine the estimates of variance in precision error to obtain a better estimate of the overall performance of each instrument. This is done automatically by the program when cases are stacked successively with measurements from the same instruments recorded in the same data fields.

D. Data Handling of Empty Cells and Outliers

One very important element of the analyses is the handling of missing data and extreme observations or outliers.

1. Missing Data and Empty Cells.

If data in one or more cells (a cell being simply one measurement by one instrument) is missing, it is necessary that something be done to assure that the data point is not considered as a reading of zero (unless zero is a valid reading).

A simple means would be to omit all other measurements of that same data point by the other instruments (one row of data). Alternately, one could eliminate all other measurements by that instrument which has an empty cell (one column of data). Provision is made in the program to do either or both of these things by means of control cards 2 and 3 of each case. One should keep in mind the deletion of an entire row or column reduces the degrees of freedom by the proportionate amount.

An alternate approach to avoid having to eliminate an entire row or column of data due to a missing data point would be to estimate the size of the missing value based on standard ANOVA procedures and substitute the estimated value for the missing data point. This would also reduce by one the degrees of freedom for the significance tests performed by the program. Since the computer program has no way of differentiating estimated values from actual readings, the degrees of freedom must be manually reduced on the computer output when observations are estimated.

2. Extreme Values and Outliers.

Since the precision estimates performed by the program are sensitive to the presence of outliers or extreme values, it is imperative that the program user deal with their presence.

Built into the program is an outlier test (a modified F-test)[3] which flags any value which is statistically determined to be an outlier at the .95 level of confidence.

Tests are performed on columns (all observations by an instrument) and on rows (observations of one data point by all instruments). If an outlier is detected it is flagged by the placement of an "I" if a column outlier and by a "P" if a row (or point) outlier. An "IP" may appear if detected by both tests. Naturally any missing data (empty cell) not omitted by the control card options (cards 2 or 3) will be flagged as outliers. In the comparative case (two known instruments versus one unknown), outliers are flagged by means of an asterisk "'*'".

It is important to note that the detection of an outlier does not dictate its automatic removal from consideration as a data point. In fact, it may be a very valuable data point. It is up to the program user to decide how to deal with the extreme values. If it is due to a typographical error or human recording error, it may be amendable. If, on the other hand, there is no rational reason to doubt the observer's authority, nor to expect the instrumentation to behave any differently on future tests, it might be wise to include the outlying observation as an acceptable data point in computing typical instrument performance.

It is suggested that on the first computer run, omit only data from empty cells and let the program flag all outliers. Then the decision can be made as to how to deal with these values on subsequent runs.

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[3] *Grubbs, F.E., "Procedures for Detecting Outlying Observations in Samples," BRL Report No. 1713, AD 781 499, April 1974.*

The only way that an entered data point may be ignored from computation is by means of control cards 2 and 3 described in the next section. Flagging a data point by the computer does not constitute the disqualification of that point, nor does the presence of a flag (I, P or *) remove the data point from the computational procedure. Removal of a data point can only be accomplished by means of the control cards.

III. EXAMPLES

A. Example 1: General Case

This case consists of 12 projectile firings with muzzle velocity measured by 9 different instruments. Measurements of the sixth occurrence have been deleted since the FBI01 (B) failed to obtain a reading (Figure 1). If the empty cell were allowed to be entered, the precision estimates would be significantly affected. This is illustrated through data gathered by the NM87A instrument (H). It failed to measure the fourth, ninth and tenth occurrences. These empty cells were entered as data points and as a result, the inflated estimate of variance in precision (est. σ^2_{eH}) = 11.6×10^4.

Also notice that the second observation made by the COMP (C) instrument has been detected as an outlier at a confidence level of 95% when compared with observations of the same occurrences by the other instruments. The same is true of six measurements by the TERMA 1 (F) instrument.

Due to the presence of the extreme values and empty cells, the precision estimates are highly different. Hence we use the case as a computer run to detect outlying observations and missing data. Shaded data indicates information deleted by control cards.

B. Example 2: General Case: Data Deleted

Based on the results of Case 1, we have decided to delete all observations from the TERMA 1 (F) and NM87A (H) due to their erratic behavior. Data point 6 is also deleted from consideration from all instruments due to a missing observation from the FBI01 (B) (Figure 2). Hence we are basing our results on measurements from 7 instruments over 11 points for a total of 77 observations. Two outliers are included since it was felt that these were real data points indicative of the performance of the instruments involved (C and G).

The analysis indicates that the COUNTER (A), FOTOCEL (E), FBI02 (D) and NM87B (I) have the best precision of measurement (lowest est. σ^2_{ei}). In fact two of the estimates have negative values (COUNTER and FOTOCEL). This is possible since the sample size is rather small and we are computing the best estimate of precision, that estimate having a chance of

EXAMPLE I: General Case

VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
ESTIMATES AND RANKING OF PRECISION OF MEASUREMENT(9 INSTRUMENTS)

	DATA POINT	COUNTER A	FBI01 B	COMP C	FBI02 D	FOTOCEL E	TERMA1 F	TERMA2 G	NM87A H	NM87B I	MEAN
	1	733.00	732.00	731.40	733.30	732.50	715.50P	730.80	732.40	733.30	730.47
	2	729.60	728.60	737.20P	729.50	729.00	724.30	728.70	725.20	729.70	729.09
	3	731.20	730.20	729.70	731.30	730.60	727.50P	730.40	730.80	731.50	730.36
	4	734.60	733.90	733.20	734.70	734.00	728.80	734.00	0.00P	734.80	652.00
	5	735.10	734.10	733.40	735.00	734.50	730.20P	734.90	733.40	735.10	733.97
DELETED	6	731.60	0.00	730.00	731.30	730.90	723.60	730.50	730.30	731.60	0.00
	7	727.90	727.10	726.40	727.80	727.30	722.20P	727.00	726.90	728.10	726.74
	8	731.70	730.70	730.00	731.70	731.10	726.50P	730.50	730.50	732.20	730.54
	9	729.10	728.40	727.80	729.10	728.50	723.80	729.00	0.00P	729.50	647.24
	10	728.10	729.10	727.00	728.20	727.60	726.60	721.80	0.00P	728.40	646.31
	11	728.80	728.00	727.40	729.00	728.20	712.70P	728.20	727.80	729.30	726.60
	12	730.50	729.70	729.20	730.60	729.90	728.40	729.00	730.60	731.20	729.90
	MEAN	730.87	730.16	730.25	730.93	730.29	724.23	729.48	530.69	731.19	
	VARIANCE	6.280	5.409	10.923	6.360	6.269	30.884	12.128	.116E 06	5.987	
	STND DEV	2.506	2.326	3.305	2.522	2.504	5.557	3.482	340.851	2.447	
	PROB ERR	1.690	1.569	2.229	1.701	1.689	3.748	2.349	229.904	1.650	

GRAND MEAN= 707.566 AVERAGE VARIANCE=12918.1467 STND DEV= .1E 03 PROB ERR=76.6623

MEASUREMENTS FOUND TO BE OUTLIERS AT A 95% CONFIDENCE LEVEL ARE FOLLOWED BY I OR P.
I INDICATES THE MEASUREMENT WAS FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF OTHER DATA POINTS BY THE SAME INSTRUMENT.
P INDICATES THE MEASUREMENT WAS FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF THE SAME DATA POINT BY OTHER INSTRUMENTS.

* INDICATES AN OUTLIER WHEN CONSIDERING THE DIFFERENCE BETWEEN INSTRUMENTS. (TWO INSTRUMENT CASE ONLY)

9 INSTRUMENTS WITH 11 DATA POINTS EACH WERE USED IN THIS ANALYSIS.

COVARIANCE MATRIX

	COUNTER A	FBI01 B	COMP C	FBI02 D	FOTOCEL E	TERMA1 F	TERMA2 G	NM87A H	NM87B I
COUNTER	0.0000	5.6759	4.8624	6.3118	6.2737	4.9128	7.7055	63.7087	6.1137
FBI01	5.6759	0.0000	4.0738	5.7171	5.6836	5.3241	6.2053	-62.4064	5.5136
COMP	4.8624	4.0738	0.0000	4.7586	4.8415	5.0016	6.5579	199.4655	4.5095
FBI02	6.3118	5.7171	4.7586	0.0000	6.3083	4.5242	7.6705	61.1543	6.1573
FOTOCEL	6.2737	5.6836	4.8415	6.3083	0.0000	4.8493	7.6418	60.4189	6.1069
TERMA1	4.9128	5.3241	5.0016	4.5242	4.8493	0.0000	4.6555	-472.1357	4.6843
TERMA2	7.7055	6.2053	6.5579	7.6705	7.6418	4.6555	0.0000	269.7988	7.5088
NM87A	63.7087	-62.4064	199.4655	61.1543	60.4189	-472.1357	269.7988	0.0000	67.7099
NM87B	6.1137	5.5136	4.5095	6.1573	6.1069	4.6843	7.5088	67.7099	0.0000
COV INCLD INST	105.5645	-24.2129	234.0707	102.6021	102.1240	-438.1839	317.7442	187.7140	108.3040
COV EXCLD INST	242.2988	372.0763	113.7926	245.2613	245.7394	786.0473	30.1192	160.1494	239.5594
EST $(SIGMA\ F1)^2$	-11.4574	24.7502	-43.5309	-10.5310	-10.4857	168.5033	-66.2327	.116E 06	-12.5334
EST (SIGMA F1)	0.00000	4.97496	0.00000	0.00000	0.00000	12.98088	0.00000	340.79007	0.00000
PRECISION RANK	4	7	2	5	6	8	1	9	3

TOTAL COVARIANCE= 347.8634 PARAMETER VARIANCE = 9.663 PARAM STD DEV. = 3.1085

EXAMPLE 2: General Case

VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
DATA GATHERED FROM MULTINATIONAL INSTRUMENTATION (WITH TWO INS. DELETED)

DATA POINT	COUNTER A	FBI01 B	COMP C	FBI02 D	FOTOCEL E	[redacted]	TERMA2 G	NM87A H DELETED	NM87B I	MEAN
1	733.00	732.00	731.40	733.30	732.50	[redacted]	730.80	732.40	733.30	732.33
2	729.60	728.60	737.20P	729.50	729.00	[redacted]	728.70	725.20	729.70	730.33
3	731.20	730.20	729.70	731.30	730.60	[redacted]	730.40	730.80	731.50	730.70
4	734.60	733.90	733.20	734.70	734.00	[redacted]	734.00	0.00	734.80	734.17
5	735.10	734.10	733.40	735.00	734.50	[redacted]	734.90	733.40	735.10	734.59
DELETED [redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	730.30	[redacted]	[redacted]
7	727.90	727.10	726.40	727.80	727.30	[redacted]	727.00	726.90	728.10	727.37
8	731.70	730.70	730.00	731.70	731.10	[redacted]	730.50	730.50	732.20	731.13
9	729.10	728.40	727.80	729.10	728.50	[redacted]	729.00	0.00	729.50	728.77
10	728.10	729.10	727.00	728.20	727.60	[redacted]	721.80P	0.00	728.40	727.17
11	728.80	728.00	727.40	729.00	728.20	[redacted]	728.20	727.80	729.30	728.41
12	730.50	729.70	729.20	730.60	729.90	[redacted]	729.00	730.60	731.20	730.01
MEAN	730.87	730.16	730.25	730.93	730.29	0.00	729.48	0.00	731.19	
VARIANCE	6.280	5.409	10.923	6.360	6.269	0.000	12.128	0.000	5.987	
STND DEV	2.506	2.326	3.305	2.522	2.504	0.000	3.482	0.000	2.447	
PROB ERR	1.690	1.569	2.229	1.701	1.689	0.000	2.349	0.000	1.650	

GRAND MEAN= 730.453 AVERAGE VARIANCE= 7.6222 STND DEV= 2.7608 PROB ERR= 1.8622

MEASUREMENTS FOUND TO BE OUTLIERS AT A 95% CONFIDENCE LEVEL ARE FOLLOWED BY I OR P.
I INDICATES THE MEASUREMENT WAS FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF OTHER DATA POINTS BY THE SAME INSTRUMENT.
P INDICATES THE MEASUREMENT WAS FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF THE SAME DATA POINT BY OTHER INSTRUMENTS.

* INDICATES AN OUTLIER WHEN CONSIDERING THE DIFFERENCE BETWEEN INSTRUMENTS. (TWO INSTRUMENT CASE ONLY)

7 INSTRUMENTS WITH 11 DATA POINTS EACH WERE USED IN THIS ANALYSIS.

COVARIANCE MATRIX

	COUNTER A	FBI01 B	COMP C	FBI02 D	FOTOCEL E	[redacted]	TERMA2 G	[redacted]	NM87B I
COUNTER	0.0000	5.6759	4.8624	6.3118	6.2737	[redacted]	7.7055	[redacted]	6.1137
FBI01	5.6759	0.0000	4.0738	5.7171	5.6836	[redacted]	6.2053	[redacted]	5.5136
COMP	4.8624	4.0738	0.0000	4.7586	4.8415	[redacted]	6.5579	[redacted]	4.5095
FBI02	6.3118	5.7171	4.7586	0.0000	6.3083	[redacted]	7.6705	[redacted]	6.1573
FOTOCEL	6.2737	5.6836	4.8415	6.3083	0.0000	[redacted]	7.6418	[redacted]	6.1069
DELETED [redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
TERMA2	7.7055	6.2053	6.5579	7.6705	7.6418	[redacted]	0.0000	[redacted]	7.5088
DELETED [redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
NM87B	6.1137	5.5136	4.5095	6.1573	6.1069	[redacted]	7.5088	[redacted]	0.0000
COV INCLD INST	36.9430	32.8694	29.6036	36.9236	36.8558	[redacted]	43.2898	[redacted]	35.9098
COV EXCLD INST	89.2545	93.3282	96.5939	89.2739	89.3417	[redacted]	82.9077	[redacted]	90.2877
EST (SIGMA E1)^2	-0.0838	0.6740	7.4944	0.0039	-0.0602	[redacted]	3.2249	[redacted]	0.0362
EST (SIGMA E1)	0.00000	0.82096	2.73760	0.06243	0.00000	[redacted]	1.79579	[redacted]	0.19014
PRECISION RANK	1	5	7	3	2	[redacted]	6	[redacted]	4

TOTAL COVARIANCE= 126.1975 PARAMETER VARIANCE = 6.009 PARAM STD DEV, = 2.4514

being above or below the real value. If est. (σ_{ei}^2) is negative, the computed est. (σ_{ei}) is set at zero on the computer printout. However, all values of est. (σ_{ei}^2), positive and negative are stored in the program to compute a pooled estimate of precision of measurement if cases involving the same instruments are stacked.

We see the estimate of the actual parameter variance (stripped of measurement error) as being est. $\sigma_x^2 = 6.009$. Hence the variance in precision of measurement is orders of magnitude less than the parameter variance point-to-point. In other words, for this example, the variation point-to-point is due largely to real product variability and not to measurement error.

C. Example 3: Comparative Case: Two "Known" Instruments Versus a Third "Unknown" Instrument

In this example, we are attempting to decide whether some "unknown" or "untested" instrument COMP (C) performs better than the average of two "known" or tried-and-tested instruments (COUNTER, (A) and FBI01 (B)) (Figures 3 and 4).

Since we want to limit our investigation to only these three instruments, all other data has been deleted. The sixth data point has also been removed from consideration since an empty cell exists.

Figure 4 lists 12 occurrences as measured simultaneously by three instruments (R, S, T). The R and the S represent the two proven standards, whereas the T represents an unknown instrument submitted for test. Figure 4 gives for firings in each combination of two instruments the individual and mean readings of the three instruments, the individual differences in measurement for the instruments taken two at a time and the computational procedures for estimating the variance in errors of measurement for each instrument, and the estimated variance of the true readings measured by each of the instruments.

These tests reveal that their variances in precision error are not significantly different ($.988 < t_{.95}$ 1.833) but their mean values are significantly different ($4.052 > t_{.95}$ 1.812) (730.87 versus 730.16). This indicates that they are in need of calibration.

Finally we proceed to compare the average performance of the "known" standard instruments with the "unknown" instrument. The second set of t-tests are performed on means and precisions. In the example, there were no significant differences (if no calibration is performed) between the combined means of the two "known" instruments (730.5) and the "unknown" instrument mean (730.2). However, the statistics show that the "unknown" instrument (COMP (C)) was significantly less precise (higher est. σ_{ei}^2)

EXAMPLE 3: Comparitive Case

VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
3 INSTRUMENT CASE(COMPARING 2 INSTRUMENTS OF KNOWN PERFORMANCE AGAINST UNKNOWN)

DATA POINT	COUNTER A	FBI01 B	COMP C	[redacted]						MEAN
1	733.00	732.00	731.40	[redacted]						732.13
2	729.60	728.60	737.20							731.80
3	731.20	730.20	729.70							730.37
4	734.60	733.90	733.20							733.90
5	735.10	734.10	733.40							734.20
DELETED [redacted]										
7	727.90	727.10	726.40							727.13
8	731.70	730.70	730.00							730.80
9	729.10	728.40	727.80							728.43
10	728.10	729.10	727.00							728.07
11	728.80	728.00	727.40							728.07
12	730.50	729.70	729.20							729.80
MEAN	730.87	730.16	730.25	0.00	0.00	0.00	0.00	0.00	0.00	
VARIANCE	6.280	5.409	10.923	0.000	0.000	0.000	0.000	0.000	0.000	
STND DEV	2.506	2.326	3.305	0.000	0.000	0.000	0.000	0.000	0.000	
PROB ERR	1.690	1.569	2.229	0.000	0.000	0.000	0.000	0.000	0.000	

GRAND MEAN= 730.427 AVERAGE VARIANCE= 7.5372 STND DEV= 2.7454 PROB ERR= 1.8518

MEASUREMENTS FOUND TO BE OUTLIERS AT A 95% CONFIDENCE LEVEL ARE FOLLOWED BY I OR P.
I INDICATES THE MEASUREMENT WAS FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF OTHER DATA POINTS BY THE SAME INSTRUMENT.
P INDICATES THE MEASUREMENT WAS FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF THE SAME DATA POINT BY OTHER INSTRUMENTS.

* INDICATES AN OUTLIER WHEN CONSIDERING THE DIFFERENCE BETWEEN INSTRUMENTS. (TWO INSTRUMENT CASE ONLY)

3 INSTRUMENTS WITH 11 DATA POINTS EACH WERE USED IN THIS ANALYSIS.

COVARIANCE MATRIX

	COUNTER A	FBI01 B	COMP C	[redacted]
COUNTER	0.0000	5.6759	4.8624	
FBI01	5.6759	0.0000	4.0738	
COMP	4.8624	4.0738	0.0000	
DELETED [redacted]				
DELETED				
DELETED				
DELETED				
DELETED				
DELETED				
COV INCLD INST	10.5383	9.7497	8.9362	
COV EXCLD INST	4.0738	4.8624	5.6759	
EST $(SIGMA^2\ E1)$	-0.1843	0.5212	7.6625	
EST (SIGMA E1)	0.00000	0.72193	2.76811	
PRECISION RANK	1	2	3	

TOTAL COVARIANCE= 14.6121 PARAMETER VARIANCE = 4.871 PARAM STD DEV. = 2.2070

EXAMPLE 3: Comparitive Case(cont'd)

VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
3 INSTRUMENT CASE(COMPARING 2 INSTRUMENTS OF KNOWN PERFORMANCE AGAINST UNKNOWN)

DATA POINT	COUNTER R(A)	FBIO1 S(B)	COMP T(C)	MEAN	R-S	S-T	T-R
1	733.0	732.0	731.4	732.13	1.0	0.6	-1.6
2	729.6	728.6	737.2	731.80	1.0	-8.6*	7.6*
3	731.2	730.2	729.7	730.37	1.0	0.5	-1.5
4	734.6	733.9	733.2	733.90	0.7	0.7	-1.4
5	735.1	734.1	733.4	734.20	1.0	0.7	-1.7
DELETED [redacted]							
7	727.9	727.1	726.4	727.13	0.8	0.7	-1.5
8	731.7	730.7	730.0	730.80	1.0	0.7	-1.7
9	729.1	728.4	727.8	728.43	0.7	0.6	-1.3
10	728.1	729.1	727.0	728.07	-1.0*	2.1	-1.1
11	728.3	728.0	727.4	728.07	0.8	0.6	-1.4
12	730.5	729.7	729.2	729.80	0.8	0.5	-1.3
				MEAN	0.709	-0.082	-0.627
				VARIANCE	0.337	8.184	7.478

EST INST VARIANCES SIGMA (E1^2) R = -0.1843 S = 0.5212 T = 7.6625

EST PARAMETER VARIANCE SIGMA^2 (E SUB X) = 4.8707
I, P, AND * FOLLOW OUTLIERS AT A .95 CONFIDENCE LEVEL.
I AND P INDICATE MEASUREMENTS FOUND TO BE OUTLIERS IN THE 3 INSTRUMENT ANALYSIS,
* FOLLOWS A VALUE FOUND TO BE AN OUTLIER WHEN COMPARED TO OTHER DATA POINTS.

DATA POINT	COUNTER R(A)	FBIO1 S(B)	COMP T(C)	Y	Z=V1	U1
1	733.0	732.0	731.4	4.0	1.0	-1.1
2	729.6	728.6	737.2	-2.8	1.0	8.1*
3	731.2	730.2	729.7	0.4	1.0	-1.0
4	734.6	733.9	733.2	7.5	0.7	-1.1
5	735.1	734.1	733.4	8.2	1.0	-1.2
DELETED [redacted]						
7	727.7	727.1	726.4	-6.0	0.8	-1.1
8	731.7	730.7	730.0	1.4	1.0	-1.2
9	729.1	728.4	727.8	-3.5	0.7	-0.9
10	728.1	729.1	727.0	-3.8	-1.0*	-1.6
11	728.8	728.0	727.4	-4.2	0.8	-1.0
12	730.5	729.7	729.2	-0.8	0.8	-0.9
MEAN	730.873	730.164	730.245	0.000	0.709	-0.273
VARIANCE	6.2802	5.4085	10.9227	23.0405	0.3349	7.7467

S(YZ)= 0.8716 S(UV)= 0.3527 R(YZ)= 0.3128 R(UV)= 0.2183

T(STNDS VAR)= 0.988 T(TEST VAR)= 8.233 T.95= 1.833 (9 D.F.)
T(STNDS BIAS)= 4.052 T(TEST BIAS)= -0.325 T.95= 1.812 (10 D.F.)

THE MEANS OF THE STANDARD INSTRUMENTS ARE SIGNIFICANTLY DIFFERENT
THE TEST INSTRUMENT IS SIGNIFICANTLY LESS PRECISE THAN THE STANDARDS

than the standard (7.66 versus -.18 and .52) since the calculated value t (8.23) exceeds the t critical at .95 with 9 degrees of freedom (1.833).

IV. DESCRIPTION OF DATA INPUT

A.

<u>Card 1</u>

Cols 1-2 - Number of data points for each instrument (integer) (total number of data cards to be read in with a minimum of 2 and maximum of 50). (The total number of data cards to be read includes data cards excluded by control card 3).

Cols 3-4 - Number of instruments making simultaneous measurements (integer) with a minimum of 2 and maximum of 10. The total number of instruments recording data includes instrumentation excluded by control card 2.

<u>Card 2</u>

Cols 1-10 - Insert the digit 1 in every column starting in column 1 out to the column indicating the number of instruments used, data for any instrument may be omitted from analyses by placing a zero rather than 1 in the column corresponding to that instrument. Example: 111101 would mean use data from all instruments except instrument 5 (field 5).

<u>Card 3</u>

Cols 1-50 - Insert the digit 1 in every column starting in column 1 out to the column indicating the number of data cards. Data for any observation may be omitted from analyses by placing a zero rather than 1 in the column corresponding to that data point. Example: 1001111111 in columns 1-10 would mean to use all data cards except those of observations 2 and 3 (data cards 2 and 3).

<u>Cards 4 and 5</u>

Cols 1-80 - Alphanumeric titles.

Card 6

Cols 1-8, 9-16, 17-24 ... 73-80 - Alphanumeric headings of names of instruments. These headings must be identical and in identical fields from case-to-case if several cases are stacked and pooled estimates of precision error are desired. New headings on this card will cause the program to pool estimates of precision on most recently processed cards with other headings.

Card 7 Data cards (up to 50 data cards, one card per data point, a maximum of ten simultaneous measurements by different instruments per point, (ten fixed point, 8 digit fields).

Cols 1-8 - Observation from instrument A of first occurrence, (fixed point)

Cols 9-16 - Observation from instrument B of first occurrence, (fixed point)

Cols 17-24 - Observation from instrument C of first occurrence, (fixed point)

. . . .

Cols 73-80 - Observation from instrument J of first occurrence, (fixed point)

Last Card Per Case

Option 1. General Case: Estimation of variances in errors of precision of j instruments ($2 \leq j \leq 10$)

Col 1 - Insert decimal point in first column.

Option 2. Comparative Case: Estimation of variances in precision of measurement and also computation of Student t to determine significance of differences in precision errors and bias errors.

I. Parametric mode (to look at all combinations of three instruments)

Cols 1-4 - Insert '***.' (three asterisks and decimal point)

II. Specific mode (to look at specific combinations)

Cols 1-4 - Insert 'ABC.' where A,B,C can be any three alphanumeric letters referring to data from instruments in those corresponding to those letters as follows (A-J):

Instrument A = data in cols 1-8
Instrument B = data in cols 9-16
Instrument C = data in cols 17-24
Instrument D = data in cols 25-32
Instrument .   data in cols ..-..
Instrument J = data in cols 73-80

The first two letters specified are considered as the standards (A, B) and the third letter (C) is considered the nonstandard test instrument. The letters need not be in alphabetical order, an asterisk (*) may be used in place of any one or two letters which then forces a repeated analyses with data for every other instrument to, in turn, be substituted for that asterisk point. Example: BC*, = BCA, BCD, BCE, ... and *C*, = ACB, ACD, ... BCA, BCD.,, DCA, DCB ...

Cases may be stacked with a limit of 50 cases per computer run.

A blank card must be placed at end of last case. This also causes the program to compute a summary page of previous cases including pooled estimates of precision error. All blank fields within the data card are considered as zero observations. If data gaps exist for one or more instruments, a zero corresponding to that instrument or occurrence on controls cards 2 or 3 causes the program to skip over incomplete data. A zero in a field omits all data for that instrument (data cards). A zero in a field of card 3 omits that observation from all instruments. Outliers can also be omitted. The program checks for outliers on all data which are not already removed by using zeros on cards 2 and/or 3.

B. Output Descriptions

Headings

Data listing (column to farthest right consists of row mean values)

Mean = mean value of data group of each instrument

Variance = sample variance of each data group

Stnd dev - standard deviation of data group of each instrument

Prob err = sample probable error of each data group

Grand mean = mean of all observations from all instruments

Average variance = arithmetic average of variances from each instrument.

Stnd dev = square root of average variance

Prob err = stnd dev * .6745

Outlier tests are performed on rows and columns of data, with decisions based on Modified F-test referenced earlier at .95 level of confidence. Outliers are only flagged. They are not deleted by the program.

Covariance matrix = sum of covariances of all combinations of two instrumens.

Cov incl inst = sum of covariances involving the instrument in that column. All other instruments considered.

Cov excld inst = sum of all other covariances listed in covariance matrix/2

Est (sigma^2_{e1}) = unbiased estimate of variance in errors of precision of instrument listed in that field. (Solution to Equation (2) or (5) depending on number of instruments.)

Precision ranking = ranks assigned to size of precision errors with 1 being the best precision (lowest σ^2_{ei} value)

Total covariance = (sum of all covariances in table)/2

Parameter variance = unbiased estimate of real values of parameter being observed. (Solution of Equation (4) or (7) depending on number of instruments.)

<u>Comparative Case</u>: Two standard instruments versus one nonstandard.

R = data from first standard instrument

S = data from second standard instrument

T = data from nonstandard instrument

Y = difference between sum of <u>mean</u> readings of R and S and sum of ith readings of R and S

Z = V1 = difference in readings between two standard instruments

U1 = difference between instrument T and average of R and S

S(YZ) = covariance of Y and Z

S(UV) = covariance of U1 and V1

R(YZ) = sample correlation coefficient of Y and Z

R(UV) = sample correlation coefficient of U and V

T(STNDS VAR) = computed t statistic from Equation (8) to determine if the standard deviation of precision errors of R and S are significantly different

T(STNDS BIAS) = computed t statistic from Equation (9) to determine if bias is significant between the two standards R and S

Statistics to determine whether standards are in close enough agreement in precision and average reading (bias):

T(TEST VAR) = computed t statistic from Equation (10) to determine if the nonstandard test instrument is operating with a precision of measurement equal to the average of the two standards.

T(TEST BIAS) = computed t statistic from Equation (11) to determine if the nonstandard test instrument is biased when compared with the average readings of the two standards.

Statistics to determine whether nonstandard third instrument (T) has a statistically different performance in precision of measurement and average reading (bias) from the two known standard instruments (R and S):

T.95 = Student t value to be used as a comparison against T(STNDS VAR) and (TEST VAR) to determine acceptance of null hypothesis. T.95 is to be used as a comparison against T(STNDS BIAS) and (TEST BIAS),

* = indicates outlier in column of data at .95 confidence level,

R-S = difference between observations of instruments R and S,

S-T = difference between observations of instruments S and T,

T-R = difference between observations of instruments T and R,

Appendix A is a complete program listing in the FORTRAN IV language.

Appendix B is a listing of the data used for the three examples shown.

ACKNOWLEDGEMENTS

I wish to thank Dr. Frank E. Grubbs for his permission to excerpt portions of Reference 1 and for his invaluable help and encouragement in this work. I also wish to thank SP-5 Drew Einhorn for his valuable assistance.

Appendix A

Program Listing

```
* MD440    OBRYON      4271 394   ESTIMATE PRECISION OF MEASUREMENT FEB 77
*       LIST(START)                                                     MAIN  1
        DIMENSION ASTR(50) $ DIMENSION N1S(50), N2S(50)                 MAIN  2
        DIMENSION PCTVEL(50), PCTVCY(50) $ DIMENSION PEP(10)            MAIN  3
        DIMENSION PES(10,50) $ DIMENSION LINSTR(10)                     MAIN  4
        DIMENSION A(50,10), SUMA(10), COV(10,10), VAR(10), COVA(10),   MAIN  5
      1  COVNA(10), PRB(10), PE(10), INSTR(10), SDEV(10), INCIST(10),  MAIN  6
      2  INCRND(50), ISTNAM(10), ISTDEL(10), GRP(80), TITLE(16),       MAIN  7
      3  IRNDEL(50) $ DIMENSION MEANA(10) $ DIMENSION IDEL(2)          MAIN  8
        DIMENSION T95(49)                                               MAIN  9
        DIMENSION Y(50), Z(50), U(50), SMT(50), TMR(50), RSTM(50)      MAIN 10
        DIMENSION APE(10), IPE(10) $ DIMENSION PEV(10,50)              MAIN 11
        DIMENSION TTV(10,10), TTL(10,10), TVL(10,10), TLL(10,10)       MAIN 12
        DIMENSION V(50), RTL(50,10), CTL(50,10) $ DIMENSION MEANS(50)  MAIN 13
        DIMENSION JPE(10) $ DIMENSION ACROSS(50)                        MAIN 14
        DIMENSION YF(50), ZF(50), UF(50), SMTF(50), TMRF(50)           MAIN 15
        DIMENSION RSV(10,10), RSL(10,10), SVL(10,10), SLL(10,10)       MAIN 16
        DIMENSION POOLV(10), NPLD(10) $ DIMENSION POOLS(10)            MAIN 17
        INTEGER R,S,T $ EQUIVALENCE (S,KS) $ INTEGER GRP                MAIN 18
        REAL MEANA, MEAN, IST $ REAL MEANS                              MAIN 19
        DATA ISTNAM /'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J'/ MAIN 20D
        DATA IDEL /'DELETED', ''/ $ DATA KY, KZ, KU, KSMT, KTMR /5*1/  MAIN 21D
        DATA T95 /6.314, 2.920, 2.353, 2.132, 2.015, 1.943, 1.895, 1.860, MAIN 22D
       11.833, 1.812, 1.796, 1.782, 1.771, 1.761, 1.753, 1.746, 1.740, 1.7MAIN 23D
       234, 1.729, 1.725, 1.721, 1.717, 1.714, 1.711, 1.708, 1.706, 1.703,MAIN 24D
       3 1.701, 1.699, 1.697, 1.695, 1.693, 1.692, 1.690, 1.689, 1.688, 1.MAIN 25D
       4687, 1.686, 1.685, 1.684, 1.683, 1.682, 1.682, 1.681, 1.680, 1.679MAIN 26D
       5, 1.679, 1.678, 1.678/ $ NSAV=1                                 MAIN 27D
        WRITE (6,148) $ ABCD='MEAN'                                     MAIN 28W
      1 READ (5,96) MAXRND,MAXIST $ IF (MAXRND.EQ.0) GO TO 4            MAIN 29R
        READ (5,91) (INCIST(I),I=1,MAXIST)                              MAIN 30R
        READ (5,91) (INCRND(I),I=1,MAXRND) $ READ (5,93) (TITLE(I),I=1,16)MAIN 31R
        READ (5,94) (INSTR(I),I=1,MAXIST)                               MAIN 32R
        DO 2 I=1,MAXRND                                                 MAIN 33
      2 READ (5,95) (A(I,J),J=1,MAXIST) $ IF (NSAV.EQ.50) GO TO 4      MAIN 34R
        MX=MAXIST $ IF (LMXIST.LT.MX) MX=LMXIST                         MAIN 35
        DO 3 I=1,MX $ IF (INSTR(I).NE.LINSTR(I)) GO TO 4                MAIN 36
      3 CONTINUE $ GO TO 18                                             MAIN 37
      4 IF (NSAV.EQ.1) GO TO 17 $ WRITE (6,149) (LINSTR(I),I=1,LMXIST)  MAIN 38W
        PCTVCY(J)=0.0 $ TOPCTV=0.0 $ N3S=0 $ N4S=0                      MAIN 39
        DO 6 J=1,NSAV $ PCTVEL(J)=0.0                                   MAIN 40
        DO 5 I=1,LMXIST $ PCTVEL(J)=PES(I,J)+PCTVEL(J)                  MAIN 41
      5 CONTINUE $ PCTVCY(J)=PCTVEL(J)/FLOAT(N1S(J))                    MAIN 42
        PCTVCY(J)=(SQRT(PCTVCY(J)))/(MEANS(J))                          MAIN 43
      6 WRITE (6,150) N1S(J),N2S(J),MEANS(J),(PES(I,J),I=1,LMXIST)      MAIN 44W
        DO 7 J=1,NSAV $ TOPCTV=PCTVCY(J)+TOPCTV $ N3S=N1S(J)+N3S        MAIN 45
        N4S=N2S(J)+N4S                                                  MAIN 46
      7 CONTINUE $ TOPCTV=((TOPCTV)/(FLOAT(NSAV)))*100.                 MAIN 47
        IF (NSAV.EQ.50) WRITE (6,151)                                   MAIN 48W
        DO 9 I=1,LMXIST $ POOLV(I)=0. $ NPLD(I)=0                       MAIN 49
        DO 8 J=1,NSAV $ IF (PES(I,J).EQ.0.) GO TO 8                     MAIN 50
        NPLD(I)=NPLD(I)+N2S(J) $ POOLV(I)=POOLV(I)+FLOAT(N2S(J))*PES(I,J) MAIN 51
      8 CONTINUE $ IF (NPLD(I).EQ.0) GO TO 9                            MAIN 52
        POOLV(I)=POOLV(I)/FLOAT(NPLD(I))                                MAIN 53
      9 CONTINUE                                                        MAIN 54
        DO 11 I=1,LMXIST $ IF (POOLV(I).LE.0.) GO TO 10                 MAIN 55
        POOLS(I)=SQRT(POOLV(I)) $ GO TO 11                              MAIN 56
     10 POOLS(I)=0.0                                                    MAIN 57
     11 CONTINUE $ WRITE (6,152) (POOLV(I),I=1,LMXIST)                  MAIN 58W
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      WRITE (6,154) (POOLS(I),I=1,LMXIST)                                MAIN 59W
      WRITE (6,153) (NPLD(I),I=1,LMXIST)                                 MAIN 60W
      WRITE (6,155) TOPCTV,LMXIST,N4S                                    MAIN 61W
      DO 15 I=1,MAXIST                                                   MAIN 62
      DO 14 J=1,NSAV $ IF (PES(I,J).GE.0.0) GO TO 12 $ GO TO 13          MAIN 63
   12 PEV(I,J)=SQRT(PES(I,J)) $ GO TO 14                                 MAIN 64
   13 PEV(I,J)=0.0                                                       MAIN 65
   14 CONTINUE                                                           MAIN 66
   15 CONTINUE $ WRITE (6,90) (LINSTR(I),I=1,LMXIST)                     MAIN 67W
      DO 16 J=1,NSAV                                                     MAIN 68
      WRITE (6,150) N1S(J),N2S(J),MEANS(J),(PEV(I,J),I=1,LMXIST)         MAIN 69W
   16 CONTINUE $ WRITE (6,152) (POOLV(I),I=1,LMXIST)                     MAIN 70W
      WRITE (6,154) (POOLS(I),I=1,LMXIST)                                MAIN 71W
      WRITE (6,153) (NPLD(I),I=1,LMXIST)                                 MAIN 72W
      WRITE (6,155) TOPCTV,LMXIST,N4S                                    MAIN 73W
   17 NSAV=0 $ LMXIST=0                                                  MAIN 74
   18 IF (MAXRND.EQ.0) STOP                                              MAIN 75
      DO 19 I=1,MAXIST                                                   MAIN 76
   19 LINSTR(I)=INSTR(I) $ IF (MAXIST.GT.LMXIST) LMXIST=MAXIST           MAIN 77
      NSAV=NSAV+1                                                        MAIN 78
      DO 20 I=1,10                                                       MAIN 79
   20 PES(I,NSAV)=0. $ RND=0.                                            MAIN 80
      DO 22 I=1,MAXRND $ Y(I)=Z(I)=U(I)=SMT(I)=TMR(I)=RSTM(I)=0.         MAIN 81
      J=INCRND(I) $ IRNDEL(I)=IDEL(J+1)                                  MAIN 82
      DO 21 J=1,MAXIST                                                   MAIN 83
   21 CTL(I,J)=RTL(I,J)=0.                                               MAIN 84
   22 IF (INCRND(I).NE.0) RND=RND+1. $ IST=0.                            MAIN 85
      DO 23 I=1,MAXIST $ J=INCIST(I) $ ISTDEL(I)=IDEL(J+1)               MAIN 86
      IF (INCIST(I).NE.0) IST=IST+1.                                     MAIN 87
      SUMA(I)=COVA(I)=VAR(I)=MEANA(I)=VAR(I)=COVNA(I)=PE(I)=PRB(I)=0.    MAIN 88
      SDEV(I)=0. $ JPE(I)=0                                              MAIN 89
      DO 23 J=1,MAXIST                                                   MAIN 90
   23 COV(I,J)=0. $ TVAR=0.                                              MAIN 91
      DO 25 I=1,MAXIST $ IF (INCIST(I).EQ.0) GO TO 25                    MAIN 92
      DO 24 J=1,MAXRND                                                   MAIN 93
   24 V(J)=A(J,I) $ CALL OUTLIR (V,INCRND,MAXRND,IND,K,VA) $ VAR(I)=VA   MAIN 94
      SDEV(I)=SQRT(VAR(I)) $ PRB(I)=.6745*SDEV(I) $ TVAR=TVAR+VA         MAIN 95
      IF (IND.NE.0) CTL(K,I)='I'                                         MAIN 96
   25 CONTINUE $ TVAR=TVAR/IST $ TSDEV=SQRT(TVAR) $ TPROBE=.6745*TSDEV   MAIN 97
      DO 27 I=1,MAXRND $ IF (INCRND(I).EQ.0) GO TO 27                    MAIN 98
      DO 26 J=1,MAXIST                                                   MAIN 99
   26 V(J)=A(I,J) $ CALL OUTLIR (V,INCIST,MAXIST,IND,K,VA)               MAIN100
      IF (IND.NE.0) RTL(I,K)='P'                                         MAIN101
   27 CONTINUE $ N1=IST+.5 $ N2=RND+.5 $ NM1=N2-1 $ NM2=NM1-1 $ TSUM=0.  MAIN102
      DO 29 I=1,MAXIST $ IF (INCIST(I).EQ.0) GO TO 29                    MAIN103
      DO 28 IRND=1,MAXRND                                                MAIN104
   28 IF (INCRND(IRND).NE.0) SUMA(I)=SUMA(I)+A(IRND,I)                   MAIN105
      TSUM=TSUM+SUMA(I) $ MEANA(I)=SUMA(I)/RND                           MAIN106
   29 CONTINUE $ MEAN=TSUM/(IST*RND) $ TCOV=0. $ KM=MAXIST-1             MAIN107
      DO 32 I=1,KM $ IF (INCIST(I).EQ.0) GO TO 32 $ K=I+1                MAIN108
      DO 31 J=K,MAXIST $ IF (INCIST(J).EQ.0) GO TO 31                    MAIN109
      DO 30 IRND=1,MAXRND                                                MAIN110
   30 IF (INCRND(IRND).NE.0) COV(I,J)=COV(I,J)+A(IRND,I)*A(IRND,J)       MAIN111
      COV(J,I)=COV(I,J)=(RND*COV(I,J)-SUMA(I)*SUMA(J))/(RND*(RND-1.))    MAIN112
      COVA(I)=COVA(I)+COV(I,J) $ COVA(J)=COVA(J)+COV(I,J)                MAIN113
   31 CONTINUE $ TCOV=TCOV+COVA(I)                                       MAIN114
   32 CONTINUE $ IF (INCIST(MAXIST).NE.0) TCOV=TCOV+COVA(MAXIST)         MAIN115
      TCOV=TCOV/2.                                                       MAIN116
      DO 36 I=1,MAXIST $ IF (INCIST(I).EQ.0) GO TO 36                    MAIN117
      COVNA(I)=TCOV-COVA(I) $ IF (N1.EQ.2) GO TO 33                      MAIN118
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      PE(I)=VAR(I)-2.*COVA(I)/(IST-1.)+2.*COVNA(I)/((IST-1.)*(IST-2.))  MAIN119
      GO TO 36                                                          MAIN120
   33 PE(I)=VAR(I)-TCOV                                                 MAIN121
      DO 34 IT=1,MAXIST $ IF (INCIST(IT).EQ.0) GO TO 34 $ IIS1=IIS2     MAIN122
      IIS2=IT                                                           MAIN123
   34 CONTINUE                                                          MAIN124
      DO 35 IT=1,MAXRND                                                 MAIN125
   35 V(IT)=A(IT,IIS1)-A(IT,IIS2)                                       MAIN126
      CALL OUTLIR (V,INCRND,MAXRND,IND,KASTR,VA)                        MAIN127
      IF (IND.NE.0) ASTR(KASTR)='*'                                     MAIN128
      J=0                                                               MAIN130
   36 PES(I,NSAV)=PE(I) $ N1S(NSAV)=N1 $ N2S(NSAV)=N2 $ MEANS(NSAV)=MEANMAIN129
      DO 37 I=1,MAXIST $ IF (INCIST(I).EQ.0) GO TO 37 $ J=J+1           MAIN131
      APE(J)=PE(I) $ IPE(J)=I                                           MAIN132
   37 CONTINUE $ CALL SORTXY (APE,IPE,J)                                MAIN133
      DO 38 I=1,J $ K=IPE(I)                                            MAIN134
   38 JPE(K)=I $ PARAV=2.*TCOV/(IST*(IST-1.))                           MAIN135
      DO 45 I=1,MAXRND $ ACRINS=0.0 $ ACROSS(I)=0.0                     MAIN136
      IF (INCRND(I).EQ.0) GO TO 39 $ GO TO 40                           MAIN137
   39 ACROSS(I)=0.0 $ GO TO 45                                          MAIN138
   40 DO 42 J=1,MAXIST $ IF (INCIST(J).NE.0) GO TO 41 $ GO TO 42        MAIN139
   41 ACROSS(I)=ACROSS(I)+A(I,J) $ ACRINS=ACRINS+1.                     MAIN140
   42 CONTINUE $ IF (ACRINS.EQ.0.0) GO TO 43 $ GO TO 44                 MAIN141
   43 ACROSS(I)=0.0 $ GO TO 45                                          MAIN142
   44 ACROSS(I)=ACROSS(I)/ACRINS                                        MAIN143
   45 CONTINUE $ WRITE (6,97) (TITLE(I),I=1,16)                         MAIN144W
      WRITE (6,98) (INSTR(I),I=1,MAXIST),ABCD                           MAIN145W
      WRITE (6,99) (ISTNAM(I),I=1,MAXIST)                               MAIN146W
      WRITE (6,100) (ISTDEL(I),I=1,MAXIST)                              MAIN147W
      DO 46 I=1,MAXRND                                                  MAIN148
   46 WRITE (6,101) IRNDEL(I),ASTR(I),I,(A(I,J),RTL(I,J),CTL(I,J),J=1,  MAIN149W
     1  MAXIST),ACROSS(I) $ WRITE (6,102) (MEANA(I),I=1,MAXIST)         MAIN150W
      WRITE (6,103) (VAR(I),I=1,MAXIST)                                 MAIN151W
      WRITE (6,104) (SDEV(I),I=1,MAXIST)                                MAIN152W
      WRITE (6,105) (PRB(I),I=1,MAXIST)                                 MAIN153W
      WRITE (6,106) MEAN,TVAR,TSDEV,TPROBE $ WRITE (6,107)               MAIN154W
      WRITE (6,108) N1,N2 $ IF (MAXRND.LE.15) GO TO 47                  MAIN155W
      WRITE (6,97) TITLE                                                MAIN156W
   47 WRITE (6,109) (INSTR(I),I=1,MAXIST)                               MAIN157W
      WRITE (6,110) (ISTNAM(I),I=1,MAXIST)                              MAIN158W
      WRITE (6,111) (ISTDEL(I),I=1,MAXIST)                              MAIN159W
      DO 48 I=1,MAXIST                                                  MAIN160
   48 WRITE (6,112) ISTDEL(I),INSTR(I),(COV(I,J),J=1,MAXIST)            MAIN161W
      WRITE (6,113) (COVA(I),I=1,MAXIST)                                MAIN162W
      WRITE (6,114) (COVNA(I),I=1,MAXIST)                               MAIN163W
      WRITE (6,115) (PE(I),I=1,MAXIST)                                  MAIN164W
      DO 51 I=1,MAXIST $ IF (PE(I)-0.) 50,50,49                         MAIN165
   49 PEP(I)=SQRT(PE(I)) $ GO TO 51                                     MAIN166
   50 PEP(I)=0.0                                                        MAIN167
   51 CONTINUE $ WRITE (6,156) (PEP(I),I=1,MAXIST) $ IF (PARAV) 52,52,53MAIN168W
   52 PARS=0.0 $ GO TO 54                                               MAIN169
   53 PARS=SQRT(PARAV)                                                  MAIN170
   54 WRITE (6,116) (JPE(I),I=1,MAXIST) $ WRITE (6,117) TCOV,PARAV,PARS MAIN171W
      ASTR(KASTR)='' $ LAST=0 $ ISTPT=1 $ ISTAT=0                       MAIN172
   55 READ (5,92) (GRP(I),I=1,80) $ ICHAR=0                             MAIN173R
   56 ICHAR=ICHAR+1 $ IF (ICHAR.EQ.81) GO TO 55 $ J=GRP(ICHAR)          MAIN174
      IF (J.EQ.0) GO TO 56                                              MAIN175
      DO 57 I=1,MAXIST $ IF (J.EQ.ISTNAM(I)) GO TO 58                   MAIN176
   57 CONTINUE $ I=0 $ IF (J.EQ.'*') GO TO 59 $ IF (J.EQ.',') GO TO 65  MAIN177
      IF (J.EQ.'.') GO TO 64 $ ISTAT=1 $ GO TO 56                       MAIN178
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58 IF (INCIST(I).EQ.0) ISTAT=1                                         MAIN179
59 GO TO (60,61,62,63), ISTPT                                          MAIN180
60 I1=I $ ISTPT=2 $ GO TO 56                                           MAIN181
61 I2=I $ IF (I2.NE.0.AND.I2.EQ.I1) ISTAT=1 $ ISTPT=3 $ GO TO 56       MAIN182
62 I3=I $ IF (I3.NE.0.AND.(I3.EQ.I1.OR.I3.EQ.I2)) ISTAT=1 $ ISTPT=4    MAIN183
   GO TO 56                                                            MAIN184
63 ISTAT=1 $ GO TO 56                                                  MAIN185
64 LAST=1 $ IF (ISTPT.EQ.1) GO TO 1                                    MAIN186
65 IF (ISTPT.NE.4) ISTAT=1 $ IF (ISTAT.EQ.0) GO TO 67 $ WRITE (6,157)MAIN187W
66 IF (LAST.NE.0) GO TO 1 $ ISTPT=1 $ ISTAT=0 $ GO TO 56               MAIN188
67 IF (I3.NE.0) GO TO 68 $ IT1=1 $ IT2=MAXIST $ GO TO 69               MAIN189
68 IT1=I3 $ IT2=I3                                                     MAIN190
69 DO 89 T=IT1,IT2 $ IF (INCIST(T).EQ.0) GO TO 89                      MAIN191
   DO 70 I=1,MAXIST                                                    MAIN192
   DO 70 J=1,MAXIST                                                    MAIN193
70 TTV(I,J)=TTL(I,J)=TVL(I,J)=TLL(I,J)=RSV(I,J)=RSL(I,J)=SVL(I,J       MAIN194
  1 )=SLL(I,J)=0. $ FT=0. $ IF (I2.NE.0) GO TO 71 $ IS1=1              MAIN195
   IS2=MAXIST $ GO TO 72                                               MAIN196
71 IS1=I2 $ IS2=I2                                                     MAIN197
72 DO 83 KS=IS1,IS2 $ IF (INCIST(S).EQ.0) GO TO 83                     MAIN198
   IF (S.EQ.T) GO TO 83 $ IF (I1.NE.0) GO TO 73                        MAIN199
   IF (I2.EQ.0) GO TO 74 $ IR1=1 $ IR2=MAXIST $ GO TO 75               MAIN200
73 IR1=I1 $ IR2=I1 $ GO TO 75                                          MAIN201
74 IR1=S+1 $ IF (IR1.GT.MAXIST) GO TO 83 $ IR2=MAXIST                  MAIN202
75 DO 83 R=IR1,IR2 $ IF (INCIST(R).EQ.0) GO TO 83                      MAIN203
   IF (R.EQ.S.OR.R.EQ.T) GO TO 83                                      MAIN204
   DO 76 I=1,MAXRND $ IF (INCRND(I).EQ.0) GO TO 76                     MAIN205
   Y(I)=A(I,R)+A(I,S)-MEANA(R)-MEANA(S) $ Z(I)=A(I,R)-A(I,S)           MAIN206
   U(I)=A(I,T)-(A(I,R)+A(I,S))/2. $ SMT(I)=A(I,S)-A(I,T)               MAIN207
   TMR(I)=A(I,T)-A(I,R) $ RSTM(I)=(A(I,R)+A(I,S)+A(I,T))/3.            MAIN208
76 CONTINUE $ CALL OUTLIR (Y,INCRND,MAXRND,IND,KY,VA)                  MAIN209
   IF (IND.NE.0) YF(KY)='*' $ CALL OUTLIR (Z,INCRND,MAXRND,IND,KZ,VA)MAIN210
   IF (IND.NE.0) ZF(KZ)='*' $ CALL OUTLIR (U,INCRND,MAXRND,IND,KU,VA)MAIN211
   IF (IND.NE.0) UF(KU)='*'                                            MAIN212
   CALL OUTLIR (SMT,INCRND,MAXRND,IND,KSMT,VA)                         MAIN213
   IF (IND.NE.0) SMTF(KSMT)='*'                                        MAIN214
   CALL OUTLIR (TMR,INCRND,MAXRND,IND,KTMR,VA)                         MAIN215
   IF (IND.NE.0) TMRF(KTMR)='*' $ S2Y=VAR(R)+VAR(S)+2.*COV(R,S)        MAIN216
   S2Z=VAR(R)+VAR(S)-2.*COV(R,S)                                       MAIN217
   S2U=VAR(T)+S2Y/4.-COV(T,S)-COV(T,R)                                 MAIN218
   S2SMT=VAR(S)+VAR(T)-2.*COV(S,T) $ S2TMR=VAR(T)+VAR(R)-2.*COV(T,R)  MAIN219
   SYZ=VAR(R)-VAR(S) $ SUV=COV(T,R)-COV(T,S)-SYZ/2.                    MAIN220
   S2RE=(S2Z-S2SMT+S2TMR)/2. $ S2SE=(S2Z+S2SMT-S2TMR)/2.               MAIN221
   S2TE=(-S2Z+S2SMT+S2TMR)/2. $ S2X=(COV(R,S)+COV(S,T)+COV(R,T))/3.    MAIN222
   RYZ=SYZ/SQRT(S2Y*S2Z) $ RUV=SUV/SQRT(S2U*S2Z)                       MAIN223
   ZMEAN=MEANA(R)-MEANA(S) $ UMEAN=MEANA(T)-(MEANA(R)+MEANA(S))/2.     MAIN224
   SMTM=MEANA(S)-MEANA(T) $ TMRM=MEANA(T)-MEANA(R)                     MAIN225
   TRSV=RYZ*SQRT(RND-2.)/SQRT(1.-RYZ*RYZ)                              MAIN226
   TRSL=ZMEAN*SQRT(RND)/SQRT(S2Z)                                      MAIN227
   TTSRV=(S2U/S2Z-.75)*SQRT(RND-2.)/SQRT(3.*(1.-RUV*RUV)*S2U/S2Z)      MAIN228
   TTSRL=UMEAN*SQRT(RND)/SQRT(S2U) $ WRITE (6,97) TITLE                MAIN229W
   WRITE (6,118) INSTR(R),INSTR(S),INSTR(T),ISTNAM(R),ISTNAM(S),       MAIN230W
  1 ISTNAM(T)                                                          MAIN231W
   WRITE (6,119) (IRNDEL(I),I,A(I,R),RTL(I,R),CTL(I,R),A(I,S),RTL(I,   MAIN232W
  1 S),CTL(I,S),A(I,T),RTL(I,T),CTL(I,T),RSTM(I),Z(I),ZF(I),SMT(I),    MAIN233W
  2 SMTF(I),TMR(I),TMRF(I),I=1,MAXRND)                                 MAIN234W
   WRITE (6,120) ZMEAN,SMTM,TMRM $ WRITE (6,121) S2Z,S2SMT,S2TMR       MAIN235W
   WRITE (6,122) S2RE,S2SE,S2TE $ WRITE (6,123) S2X                    MAIN236W
   WRITE (6,124) N1 $ IF (MAXRND-12) 78,78,77                          MAIN237W
77 WRITE (6,97) TITLE                                                  MAIN238W
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78 WRITE (6,125) INSTR(R),INSTR(S),INSTR(T),ISTNAM(R),ISTNAM(S),       MAIN239W
  1  ISTNAM(T)                                                         MAIN240W
   WRITE (6,126) (IRNDEL(I),I,A(I,R),RTL(I,R),CTL(I,R),A(I,S),RTL(I,  MAIN241W
  1  S),CTL(I,S),A(I,T),RTL(I,T),CTL(I,T),Y(I),YF(I),Z(I),ZF(I),U(I),MAIN242W
  2  UF(I),I=1,MAXRND) $ ZERO=0.0                                      MAIN243W
   WRITE (6,127) MEANA(R),MEANA(S),MEANA(T),ZERO,ZMEAN,UMEAN           MAIN244W
   WRITE (6,128) VAR(R),VAR(S),VAR(T),S2Y,S2Z,S2U                      MAIN245W
   WRITE (6,129) SYZ,SUV,RYZ,RUV                                       MAIN246W
   WRITE (6,130) TRSV,TTSRV,T95(NM2),NM2,TRSL,TTSRL,T95(NM1),NM1       MAIN247W
   TTV(S,R)=TTV(R,S)=TTSRV $ TTL(S,R)=TTL(R,S)=TTSRL                   MAIN248
   RSV(R,S)=RSV(S,R)=TRSV $ RSL(R,S)=RSL(S,R)=TRSL                     MAIN249
   IF (ABS(TRSV).LT.T95(NM2)) GO TO 79 $ WRITE (6,131)                 MAIN250W
   SVL(R,S)=SVL(S,R)='*'                                               MAIN251
79 IF (ABS(TRSL).LT.T95(NM1)) GO TO 80 $ WRITE (6,132)                 MAIN252W
   SLL(R,S)=SLL(S,R)='*'                                               MAIN253
80 IF (ABS(TTSRV).LE.T95(NM2)) GO TO 82                                MAIN254
   IF (S2TE.LE.(S2RE+S2SE)/2.) GO TO 81 $ TVL(R,S)=TVL(S,R)='-'        MAIN255
   WRITE (6,134) $ GO TO 82                                            MAIN256W
81 TVL(R,S)=TVL(S,R)='+' $ WRITE (6,133)                               MAIN257W
82 IF (ABS(TTSRL).LE.T95(NM1)) GO TO 83 $ WRITE (6,135)                MAIN258W
   TLL(R,S)=TLL(S,R)='*'                                               MAIN259
   YF(KY)=ZF(KZ)=UF(KU)=SMTF(KSMT)=TMRF(KTMR)=0. $ WRITE (6,124) N1    MAIN260W
83 CONTINUE $ IF (I1.NE.0) GO TO 66 $ IF (I2.NE.0) GO TO 66            MAIN261
   IF (FT.NE.0.) GO TO 86 $ FT=1. $ WRITE (6,97) TITLE                 MAIN262W
   WRITE (6,136) $ WRITE (6,144) T95(NM2),NM2                          MAIN263W
   WRITE (6,138) (INSTR(I),I=1,MAXIST)                                 MAIN264W
   WRITE (6,139) (ISTDEL(I),I=1,MAXIST)                                MAIN265W
   DO 84 I=1,MAXIST                                                    MAIN266
84 WRITE (6,140) ISTDEL(I),INSTR(I),(RSV(I,J),SVL(I,J),J=1,MAXIST)     MAIN267W
   WRITE (6,146) $ WRITE (6,145) T95(NM1),NM1                          MAIN268W
   WRITE (6,138) (INSTR(I),I=1,MAXIST)                                 MAIN269W
   WRITE (6,139) (ISTDEL(I),I=1,MAXIST)                                MAIN270W
   DO 85 I=1,MAXIST                                                    MAIN271
85 WRITE (6,140) ISTDEL(I),INSTR(I),(RSL(I,J),SLL(I,J),J=1,MAXIST)     MAIN272W
   WRITE (6,147)                                                       MAIN273W
86 WRITE (6,97) TITLE $ WRITE (6,136)                                  MAIN274W
   WRITE (6,137) INSTR(T),T95(NM2),NM2                                 MAIN275W
   WRITE (6,138) (INSTR(I),I=1,MAXIST)                                 MAIN276W
   WRITE (6,139) (ISTDEL(I),I=1,MAXIST)                                MAIN277W
   DO 87 I=1,MAXIST                                                    MAIN278
87 WRITE (6,140) ISTDEL(I),INSTR(I),(TTV(I,J),TVL(I,J),J=1,MAXIST)     MAIN279W
   WRITE (6,141) $ WRITE (6,142) T95(NM1),NM1,(INSTR(I),I=1,MAXIST)    MAIN280W
   WRITE (6,139) (ISTDEL(I),I=1,MAXIST)                                MAIN281W
   DO 88 I=1,MAXIST                                                    MAIN282
88 WRITE (6,140) ISTDEL(I),INSTR(I),(TTL(I,J),TLL(I,J),J=1,MAXIST)     MAIN283W
   WRITE (6,143)                                                       MAIN284W
89 CONTINUE $ GO TO 66                                                 MAIN285
C                                                                      MAIN286
90 FORMAT (1H1,12X,49HSUMMARY OF STANDARD DEVIATION OF PRECISION ERROMAIN287
  1RS//(6X,4HINST,4X,6HPOINTS,6X,4HMEAN,10(2X,A8)))                    MAIN288
91 FORMAT (80I1)                                                       MAIN289
92 FORMAT (80A1)                                                       MAIN290
93 FORMAT (8A10)                                                       MAIN291
94 FORMAT (10A8)                                                       MAIN292
95 FORMAT (10F8.1)                                                     MAIN293
96 FORMAT (2I2)                                                        MAIN294
97 FORMAT (1H1,15X,8A10,/,1H ,15X,8A10)                                MAIN295
98 FORMAT (1H0,7X,4HDATA,10(2X,A8,1X),2X,A8)                           MAIN296
99 FORMAT (6X,5HPOINT,10(7X,A1,3X))                                    MAIN297
100 FORMAT (11X,10(2X,A8,1X))                                          MAIN298
```

```
101 FORMAT (A8,A1,I2,10(F9.2,2A1),F9.1)
102 FORMAT (1H0,7X,4HMEAN,10(F9.2,2X))
103 FORMAT (3X,8HVARIANCE,10(F9.3,2X))
104 FORMAT (3X,8HSTND DEV,10(F9.3,2X))
105 FORMAT (3X,8HPROB ERR,10(F9.3,2X))
106 FORMAT (15H0   GRAND MEAN=,F10.3,20H   AVERAGE VARIANCE=,F10.4,12H
   1   STND DEV=,F7.4,12H   PROB ERR=,F7.4)
107 FORMAT (84H0MEASUREMENTS FOUND TO BE OUTLIERS AT A 95  CONFIDENCE
   1LEVEL ARE FOLLOWED BY I OR P./130H I INDICATES THE MEASUREMENT WAS
   2 FOUND TO BE AN OUTLIER WHEN COMPARED TO MEASUREMENTS OF OTHER DAT
   3A POINTS BY THE SAME INSTRUMENT./73H P INDICATES THE MEASUREMENT W
   4AS FOUND TO BE AN OUTLIER WHEN COMPARED TO ,57HMEASUREMENTS OF THE
   5 SAME DATA POINT BY OTHER INSTRUMENTS./103H0* INDICATES AN OUTLIER
   6 WHEN CONSIDERING THE DIFFERENCE BETWEEN INSTRUMENTS. (TWO INSTRUM
   7ENT CASE ONLY))
108 FORMAT (1H0,I2,18H INSTRUMENTS WITH ,I2,45H DATA POINTS EACH WERE
   1USED IN THIS ANALYSIS.)
109 FORMAT (1H0,40X,17HCOVARIANCE MATRIX/,1H0,16X,10(2X,A8))
110 FORMAT (14X,10(9X,A1))
111 FORMAT (16X,10(2X,A8))
112 FORMAT (2A8,10(1X,F9.4))
113 FORMAT (17H0  COV INCLD INST,10(1X,F9.4))
114 FORMAT (17H0  COV EXCLD INST,10(1X,F9.4))
115 FORMAT (12X,1H2/17H   EST (SIGMA E1),10(1X,F9.4))
116 FORMAT (17H0  PRECISION RANK,10(I8,2X))
117 FORMAT (22H0    TOTAL COVARIANCE=,F10.4,4X,20HPARAMETER VARIANCE =
   1F10.3,2X,16HPARAM STD DEV. =,F8.4)
118 FORMAT (12H        DATA,3(1X,A8,2X),4X,4HMEAN,10X,3HR-S,8X,3HS-T,8
   1X,3HT-R,/6X,5HPOINT,5X,2HR(,A1,1H),7X,2HS(,A1,1H)7X,2HT(,A1,1H)/)
119 FORMAT ((A8,1X,I2,3(F9.1,2A1),F9.2,2X,3(F10.1,A1)))
120 FORMAT (1H0,51X,4HMEAN,3F11.3)
121 FORMAT (47X,8HVARIANCE,3F11.3)
122 FORMAT (1H0,28X,1H2/37H EST INST VARIANCES SIGMA (E1 )   R =,F11.4
   1,6H   S =,F11.4,6H   T =,F11.4)
123 FORMAT (1H0,37X,1H2/41H0EST PARAMETER VARIANCE SIGMA (E SUB X) =,F
   110.4)
124 FORMAT (55H I, P, AND * FOLLOW OUTLIERS AT A .95 CONFIDENCE LEVEL.
   1/59H I AND P INDICATE MEASUREMENTS FOUND TO BE OUTLIERS IN THE ,I2
   2,21H INSTRUMENT ANALYSIS.,/77H * FOLLOWS A VALUE FOUND TO BE AN OU
   3TLIER WHEN COMPARED TO OTHER DATA POINTS.)
125 FORMAT (1H0,7X,4HDATA,3(1X,A8,2X),9X,1HY,7X,4HZ=V1,9X,2HU1,/,6X,5H
   1POINT,5X,2HR(,A1,1H),7X,2HS(,A1,1H),7X,2HT(,A1,1H))
126 FORMAT ((A8,I3,3(F9.1,2A1),3(F10.1,A1)))
127 FORMAT (1H0,7X,4HMEAN,3(F9.3,2X),3(F10.3,1X))
128 FORMAT (3X,8HVARIANCE,3(F9.4,2X),3(F10.4,1X))
129 FORMAT (10H0   S(YZ)=,F8.4,9H   S(UV)=,F8.4,9H   R(YZ)=,F8.4,9H
   1R(UV)=,F8.4)
130 FORMAT (17H0   T(STNDS VAR)=,F8.3,15H   T(TEST VAR)=,F8.3,8H   T.9
   15=,F8.3,2H (,I2,6H D.F.)/17H   T(STNDS BIAS)=,F8.3,15H  T(TEST BIA
   2S)=,F8.3,8H   T.95=,F8.3,2H (,I2,6H D.F.)/)
131 FORMAT (70H THE VARIANCES OF THE STANDARD INSTRUMENTS ARE SIGNIFIC
   1ANTLY DIFFERENT)
132 FORMAT (66H THE MEANS OF THE STANDARD INSTRUMENTS ARE SIGNIFICANTL
   1Y DIFFERENT)
133 FORMAT (69H THE TEST INSTRUMENT IS SIGNIFICANTLY MORE PRECISE THAN
   1 THE STANDARDS)
134 FORMAT (69H THE TEST INSTRUMENT IS SIGNIFICANTLY LESS PRECISE THAN
   1 THE STANDARDS)
135 FORMAT (69H THE MEAN OF THE TEST INSTRUMENT DIFFERS SIGNIFICANTLY
   1FROE STANDARDS)
```

```
136 FORMAT (16HOT-VALUE SUMMARY)                                        MAIN359
137 FORMAT (79HOT-VALUES FOR TEST INSTRUMENT HAVING VARIANCE SIGNIFICAMAIN360
   1NTLY FROM THE STANDARDS./17H TEST INSTRUMENT ,A8,10X,5HT.95=,F8.3,MAIN361
   22H (,I2,6H D.F.))                                                   MAIN362
138 FORMAT (1H0,10X,5HSTNDS,10(2X,A8))                                  MAIN363
139 FORMAT (16X,10(2X,A8))                                              MAIN364
140 FORMAT (2A8,10(F9.3,A1))                                            MAIN365
141 FORMAT (68HO+ INDICATES THE TEST INSTRUMENT IS MORE PRECISE THAN TMAIN366
   1HE STANDARDS./68H - INDICATES THE TEST INSTRUMENT IS LESS PRECISE MAIN367
   2THAN THE STANDARDS.)                                                MAIN368
142 FORMAT (/////54HOT-VALUES FOR TEST INSTRUMENT HAVING SIGNIFICANT BMAIN369
   1IAS.,/6H T.95=,F8.3,2H (,I2,6H D.F.),/1H0,7X,8H    STNDS,10(2X,A8))MAIN370
143 FORMAT (85HO* INDICATES THERE IS SIGNIFICANT BIAS BETWEEN THE TESTMAIN371
   1 INSTRUMENT AND THE STANDARDS.)                                     MAIN372
144 FORMAT (65HOT-VALUES FOR STANDARDS HAVING SIGNIFICANTLY DIFFERENT MAIN373
   1VARIANCES./6H T.95=,F8.3,2H (,I2,6H D.F.))                          MAIN374
145 FORMAT (/////48HOT-VALUE FOR SIGNIFICANT BIAS BETWEEN STANDARDS./6MAIN375
   1H T.95=,F8.3,2H (,I2,6H D.F.))                                      MAIN376
146 FORMAT (87HO* INDICATES THERE IS A SIGNIFICANT DIFFERENCE IN THE VMAIN377
   1ARIANCES OF THESE TWO STANDARDS.)                                   MAIN378
147 FORMAT (69HO* INDICATES THERE IS A SIGNIFICANT BIAS BETWEEN THESE MAIN379
   1TWO STANDARDS.)                                                     MAIN380
148 FORMAT (61HOTHE THEORY FOR THE THREE INSTRUMENT ANALYSIS IS PRESENMAIN381
   1TED IN/34HO      ARDC TECHNICAL REPORT NO. 11/57H      THE STATISTIMAIN382
   2CAL COMPARISON OF MEASURING INSTRUMENTS/30H      F. E. GRUBBS AND MAIN383
   3J. F. O5HBRYON/15H      JULY 1971)                                  MAIN384
149 FORMAT (1H1,12X,1H2/25H    EST (SIGMA E1) SUMMARY//(6X,4HINST,4X,6HMAIN385
   1POINTS,6X,4HMEAN,10(2X,A8)))                                        MAIN386
150 FORMAT (2I10,11F10.5)                                               MAIN387
151 FORMAT (59H THERE IS NOT ENOUGH STORAGE TO SUMMARIZE MORE THAN 50 MAIN388
   1RUNS)                                                               MAIN389
152 FORMAT (30HOPOOLED VARIANCE IN PRECIS ERR,1X,10F10.5)               MAIN390
153 FORMAT (24H NUMBER OF POINTS POOLED5X,10I10)                        MAIN391
154 FORMAT (30HOPOOLED STND DEV IN PRECIS ERR,1X,10F10.5)               MAIN392
155 FORMAT (88HOSTANDARD DEVIATION OF PRECISION ERRORS AS A PERCENT OFMAIN393
   1 LEVEL OF CHARACTERISTIC (POOLED)/11H  PERCENT =,F8.5,9H BASED ON,MAIN394
   2I5,15H INSTRUMENTS ON,I5,11H OCCASIONS.)                            MAIN395
156 FORMAT (17H    EST (SIGMA E1),10F10.5)                              MAIN396
157 FORMAT (50H1IMPROPERLY DESCRIBED GROUP FOR 3 INSTRUMENT CASE.)      MAIN397
    END                                                                 MAIN398-
    SUBROUTINE OUTLIR (V,I,N,IND,K,VAR)                                 ****   1
    DIMENSION V(N),I(N),R95(50)                                         OUTLIR 2
    DATA R95/0.,0.,                                                     OUTLIR 3
   C          .0027,.0494,.1270,.2032,.2696,.3261,.3742,.4154,.4511,    OUTLIR 4
   C          .4822,.5097,.5340,.5559,.5755,.5933,.6095,.6243,.6379,    OUTLIR 5
   C          .6504,.6621,.6728,.6829,.6923,.7008,.7087,.7161,          OUTLIR 6
   C          .7232,.7300,.7365,.7427,.7486,.7543,.7597,.7649,          OUTLIR 7
   C          .7698,.7744,.7788,.7829,.7869,.7908,.7946,.7982,          OUTLIR 8
   C          .8017,.8050,.8082,.8113,.8143,.8172/                      OUTLIR 9
    AI=SUM=VDM=SS=SUM1=SS1=IND=0$ DO 1 J=1,N$ IF (I(J).EQ.0) GO TO 1    OUTLIR10
    AI=AI+1.$ SUM=SUM+V(J)                                              OUTLIR11
  1 CONTINUE$ VB=SUM/AI$ DO 3 J = 1,N$ IF (I(J).EQ.0) GOTO 3            OUTLIR12
    VD = ABS(V(J)-VB)$ IF (VD.LT.VDM) GO TO 2$ K=J$ VDM=VD              OUTLIR13
  2 SS=SS+V(J)*V(J)                                                     OUTLIR14
  3 CONTINUE$ SUM1=SUM-V(K)$ SS1=SS-V(K)*V(K)$ SS=SS-    SUM*SUM/AI     OUTLIR15
    SS1=SS1-         SUM1*SUM1/(AI-1.)$ VAR=SS/     (AI-1.)             OUTLIR16
    IF (SS.EQ.0.) RETURN$ R = SS1/SS$ L=AI+.5$ IF(L.LT.3)RETURN         OUTLIR17
    IF (R.GT.R95(L)) RETURN$ IND=1$ RETURN$ END                         OUTLIR18
```

```
      SUBROUTINE SORTXY(X,Y,N)
      DIMENSION X(N),Y(N)
      M=N
    1 M=M/2
      IF(M.EQ.0)RETURN
      K=N-M+1
      J=1
    2 I=J
    3 L=I+M
      IF(X(I).GT.X(L))GOTO 5
    4 J=J+1
      IF(J-K)2,1,1
    5 T=X(L)
      X(L)=X(I)
      X(I)=T
      T=Y(L)
      Y(L)=Y(I)
      Y(I)=T
INTERCHANGE THE I-TH AND L-TH ELEMENTS OF ADDITIONAL VECTORS.
    6 I=I-M
      IF(I)4,4,3
      END
```

Appendix B

Sample Data Input

```
*     DATA
12 9
111111111
111110111111
VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
ESTIMATES AND RANKING OF PRECISION OF MEASUREMENT(9 INSTRUMENTS)
COUNTER  FBI01   COMP    FBI02   FOTOCEL  TERMA1  TERMA2  NM87A   NM87B
 733.0   732.0   731.4   733.3   732.5   715.50  730.8   732.4   733.3
 729.6   728.6   737.2   729.5   729.0   724.30  728.7   725.2   729.7
 731.2   730.2   729.7   731.3   730.6   727.50  730.4   730.8   731.5
 734.6   733.9   733.2   734.7   734.0   728.80  734.0   000.0   734.8
 735.1   734.1   733.4   735.0   734.5   730.20  734.9   733.4   735.1
 731.6     0.0   730.0   731.3   730.9   723.60  730.5   730.3   731.6
 727.9   727.1   726.4   727.8   727.3   722.20  727.0   726.9   728.1
 731.7   730.7   730.0   731.7   731.1   726.50  730.5   730.5   732.2
 729.1   728.4   727.8   729.1   728.5   723.80  729.0   000.0   729.5
 728.1   729.1   727.0   728.2   727.6   726.6   721.80  000.0   728.4
 728.8   728.0   727.4   729.0   728.2   712.70  728.2   727.8   729.3
 730.5   729.7   729.2   730.6   729.9   728.4   729.0   730.6   731.2
.
12 9
111110101
111110111111
VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
DATA GATHERED FROM MULTINATIONAL INSTRUMENTATION (WITH TWO INS. DELETED)
COUNTER  FBI01   COMP    FBI02   FOTOCEL  TERMA1  TERMA2  NM87A   NM87B
 733.0   732.0   731.4   733.3   732.5   715.50  730.8   732.4   733.3
 729.6   728.6   737.2   729.5   729.0   724.30  728.7   725.2   729.7
 731.2   730.2   729.7   731.3   730.6   727.50  730.4   730.8   731.5
 734.6   733.9   733.2   734.7   734.0   728.80  734.0   000.0   734.8
 735.1   734.1   733.4   735.0   734.5   730.20  734.9   733.4   735.1
 731.6     0.0   730.0   731.3   730.9   723.60  730.5   730.3   731.6
 727.9   727.1   726.4   727.8   727.3   722.20  727.0   726.9   728.1
 731.7   730.7   730.0   731.7   731.1   726.50  730.5   730.5   732.2
 729.1   728.4   727.8   729.1   728.5   723.80  729.0   000.0   729.5
 728.1   729.1   727.0   728.2   727.6   726.6   721.80  000.0   728.4
 728.8   728.0   727.4   729.0   728.2   712.70  728.2   727.8   729.3
 730.5   729.7   729.2   730.6   729.9   728.4   729.0   730.6   731.2
.
12 9
111000000
111110111111
VELOCIMETER DATA , WAALSDORP PROVING GROUND SERIES 13
3 INSTRUMENT CASE(COMPARING 2 INSTRUMENTS OF KNOWN PERFORMANCE AGAINST UNKNOWN)
COUNTER  FBI01   COMP    FBI02   FOTOCEL  TERMA1  TERMA2  NM87A   NM87B
 733.0   732.0   731.4   733.3   732.5   715.50  730.8   732.4   733.3
 729.6   728.6   737.2   729.5   729.0   724.30  728.7   725.2   729.7
 731.2   730.2   729.7   731.3   730.6   727.50  730.4   730.8   731.5
 734.6   733.9   733.2   734.7   734.0   728.80  734.0   000.0   734.8
 735.1   734.1   733.4   735.0   734.5   730.20  734.9   733.4   735.1
 731.6     0.0   730.0   731.3   730.9   723.60  730.5   730.3   731.6
 727.9   727.1   726.4   727.8   727.3   722.20  727.0   726.9   728.1
 731.7   730.7   730.0   731.7   731.1   726.50  730.5   730.5   732.2
 729.1   728.4   727.8   729.1   728.5   723.80  729.0   000.0   729.5
 728.1   729.1   727.0   728.2   727.6   726.6   721.80  000.0   728.4
 728.8   728.0   727.4   729.0   728.2   712.70  728.2   727.8   729.3
 730.5   729.7   729.2   730.6   729.9   728.4   729.0   730.6   731.2
ABC.
```

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